Sparsity-Mediated Microscopy

Breaking Abbes Limit by Numerical Means

We present the experimental reconstruction of sub-wavelength features from the far-field of sparse optical objects. We show that it is sufficient to know that the object is sparse, and only that, and recover 100 nm features with the resolution of 30 nm, for an illuminating wavelength of \( \lambda = 532 \) nm. Our technique works in real-time, requires no scanning, and can be implemented in all existing microscopes - optical and non-optical.

Background & Introduction

A fundamental restriction of optical imaging is given by the diffraction limit, stating that the maximal recoverable resolution is set to half of the optical wavelength \( \lambda \). This is a direct result of the evanescent nature of all plane-waves associated with spatial frequencies exceeding \( 1/\lambda \) [1]. Consequently, spatial frequencies higher than \( 1/\lambda \) are lost, even after short propagation distances of just a few wavelengths. Hence, using optical means to resolve sub-wavelength features from the far-field is virtually impossible. Reaching beyond the sub-wavelength barrier is a subject of intense research. A most useful approach is the Scanning Near-field Optical Microscope (SNOM) [2] which probes the EM field adjacent to the illuminated sample in the „near field“ zone. Although the SNOM became a widely used method, its major drawback is the need to scan the sample point by point, preventing its use from real time applications.

Alternatively, using the "hyperlens" made of negative-index metamaterials can transform the evanescent modes into propagating ones, enabling direct imaging of sub-wavelength information [3]. However, albeit offering a great promise, negative-index materials are currently severely restricted by high material loss, stringent fabrication requirements and the need to position them in the near-field of the sample. Distributing smaller-than-wavelength fluorescent particles on the sample, exciting them in various (linear and nonlinear) means, repeating the experiments multiple times and ensemble-averaging, constitutes another approach. But this method is not real-time either [4].

A more recent idea employs super-oscillations for sub-wavelength imaging, but this
method still requires scanning, either in the near-field or in the plane where the super-oscillations are generated [5].

Apart from these "hardware solutions", several attempts have been made to extrapolate the frequency content above the cut-off frequency dictated by the diffraction limit. However, all of these extrapolation methods are extremely sensitive to noise in the measured data and the assumptions made on the prior knowledge on the information. As such, they have all failed in recovering optical sub-wavelength information [1].

In our work, we show that sub-wavelength information can be recovered from the far-field of an optical image, with the only prior knowledge being that the image is sparse [6, 7]. The idea is based on recent compressed sensing (CS) techniques [8], which are generically used for efficient sampling of data. These methods are extremely robust to noise in the measured data. Their only condition is that the information is sparse.

**Explanation & Intuition**

The underlying logic is that sparsely represented signals hold a very limited number of degrees of freedom, since only a small fraction of their coefficients (in the particular basis representation in which the signal is sparse) are non-zero. This enables to separate two subspaces of the basic functions: the one carrying information of our signal while the other carries almost none. The aim of our approach is to automatically identify the first subspace and ignore the second.

A key observation is that the reconstruction process corresponds to reconstructing a signal from the limited set of measurements from the low spatial frequencies. Hence, we need to compensate for the lost high spatial frequencies by assuming additional prior information on the signal, which is its sparsity. We define $\beta$ as the sparsity level: the relative fraction of non-zero elements in the sparsifying basis of
the signal. Since each non-zero component possesses two degrees of freedom - one for its location and the second for its amplitude, one should perform at least a 2\(\beta\) fraction of the total number of possible measurements, in order to reconstruct the signal. To gain more intuition in which basis the measurements should be performed, let us consider measurements performed in the same basis in which the signal is sparse. Then, the vast majority of the measurements would be zero and cannot provide information about the true signal. In fact we would have to carry out almost all measurements in that basis in order to ensure exact reconstruction.

Instead, we wish to choose the measurement basis such that each measurement of any projection contains information about the signal. This can be achieved by requiring that each measurement basis function has low correlation with each signal basis function. A highly uncorrelated pair of bases obeys an uncertainty principle, preventing a signal from being sparse in both bases and ensuring that, if the signal is sparse in one of the bases, it will be very spread in the other. Therefore almost each projection will yield a non-zero informative measurement in the non-sparse basis. Examples of maximally uncorrelated bases are the spatial and Fourier domains: a highly sparse signal (a single delta function) is Fourier-transformed into a spread function covering the entire spectrum. In our sub-wavelength optical setting, we are restricted to measuring only low frequencies. Accordingly, these will be sufficient to recover the signal if it is sparse in a real-space basis that is uncorrelated with the Fourier basis. There exist several algorithms for retrieving subwavelength information following our logic. We refer the interested reader to our recent publications [6, 7].

**Experiments**

For the experimental demonstration of our results, we fabricated a Star of David, consisting of 30 sub-wavelength holes, with a diameter of 100 nm each, spaced by 100 nm. Figure 1a shows an SEM image of this sample. When illuminating such a sample with monochromatic light at \(\lambda = 532\) nm, this sample practically represents a one-dimensional sub-wavelength problem, and is therefore well suited to demonstrate our reconstruction technique. In our microscope, the observed image is very small and highly blurred, as shown in figure 1b. When imaging the optical far-field (fig. 1c) it is evident that it covers much more pixels than the blurred image, therefore facilitating a much higher number of meaningful measurements (since each pixel corresponds to one measurement). Taking the data from this image and searching for the sparsest solution reproducing it, we reconstruct a Star of David, as presented in figure 1d.

**Discussion and Conclusions**
In this work, we have described a technique facilitating real-time reconstruction of sub-wavelength features, at an unprecedented resolution for single-shot experiments. The method relies on prior knowledge - that the sample is sparse in a known basis (spatial near-field, in the examples here), and only that. It is hereby important to note that most natural and artificial objects are sparse, in some known basis. The information does not necessarily have to be sparse in real space: it can be sparse in any mathematical basis whose relation to the measurement basis is known, e.g. the wavelet basis or the second derivative of the sample's profile (fig. 2).

In all of these cases, our technique can provide a major improvement by "looking beyond the resolution limit" in a single-shot experiment. Since our approach is purely algorithmic, it can be applied to every optical microscope as a simple computerized image processing tool, delivering results in real time with practically no additional hardware. Our technique is very general, and can be extended also to other, non-optical, microscopes, such as atomic force microscope, scanning-tunnelling microscope, magnetic microscopes, and other imaging systems. We believe that the microscopy technique presented here holds the promise to revolutionize the world of microscopy with just minor adjustments to current technology: sparse sub-wavelength images could be recovered by making efficient use of their available degrees of freedom.

References

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